

# Serial Data Link Analysis

Application Note

## Abstract:

Analysis of serial data channels is provided by a software application package called SDLA Visualizer which runs directly on the oscilloscope and provides extensive capability for computing embed and de-embed filters for real-time measurement and simulation. It controls the oscilloscope and creates live waveforms with these filters which can be further processed with CTLE or FFE and DFE equalization techniques, including IBIS-AMI models. SDLA automatically controls DPOJET Timing, Jitter, and Eye Analysis software to perform measurements and eye diagrams for the various test points in the SDLA channel model. This paper focuses on the **embed** and **de-embed** operations of SDLA and provides measurement examples.

## SDLA Overview

The main menu of the SDLA Visualizer is shown in Figure 1 and includes a signal processing path for de-embedding the fixture, cables, probe, and the oscilloscope used to acquire the waveform from the DUT. This path consists of the Tx and De-embed blocks. The processing of the signal begins with the oscilloscope acquired waveforms labeled as Ch1 and Ch2. The user loads S-parameters into the de-embed block path to represent the characteristics of the measurement system. SDLA then computes filters that are applied to the acquired waveforms to obtain the waveforms at various test points back to the Thevenin equivalent voltage of the Tx model.

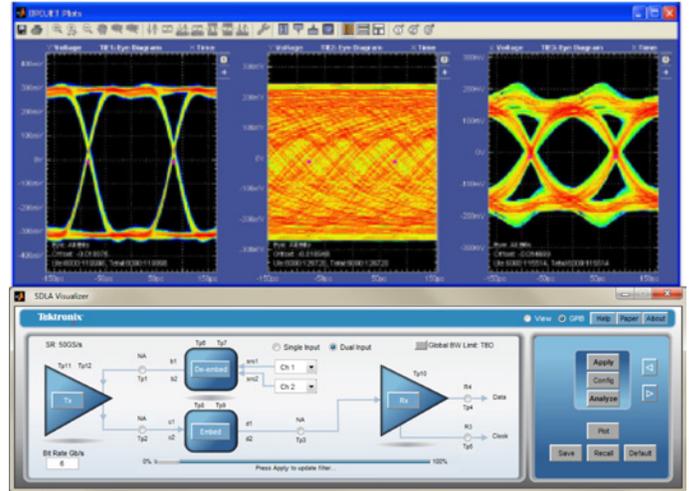


Figure 1. SDLA, Serial Data Link Analysis application.

Once the Thevenin equivalent voltage is obtained it is applied to the simulation path that consists of the Tx, Embed, and Rx block. This path is modeled with S-parameters up to the Rx blocks.

The Rx block allows the user to specify a CTLE (Continuous Time Linear Equalizer), FFE (Feed Forward Equalizer), or DFE (Decision Feedback Equalizer), that will be applied to the waveform. This creates the data waveform and the recovered clock waveform and stores them into REF3 and REF4 of the oscilloscope. The user may alternatively choose to provide an IBIS-AMI model for this block.

## Summary of SDLA System Architecture:

1. **Inputs to the System** are single or dual acquired waveforms.
2. **Measurement Path** consists of S-parameter or RLC models for Tx, and De-embed.
  - Models include: 1-port, 2-port, or 4-port S-parameters, RLC, lossless transmission line, 3-port high Z probes, interposer, SMA probes, FIR filters, and transfer functions. It also includes 6-port or dual 3-port interposer models at the probe tip, when a high impedance probe is in use.
3. **Simulation Path:** This consists of Tx, Embed, and Rx, and blocks.
  - Models include: 1-port, 2-port, and 4-port S-parameters, RLC, lossless transmission line, 3-port high Z probes, FIR filters, and transfer functions.
4. **Output from the System** are test point waveforms live on the oscilloscope screen. These test points may be full differential and may consist of up to four waveforms each. These are:
  - A mode: Line A waveform
  - B mode: Line B waveform
  - A – B: Differential mode waveform
  - (A+B)/2 : Common mode waveform
5. **Analyze:** Test point waveform are input to DPOJET data analysis package for timing, jitter, and eye analysis measurements and plots.
6. **Verification:** Extensive plotting capabilities are provided to verify and analyze S-parameter data and transfer functions of test points.
  - S-parameter plots in matrix layout format
    - i. Magnitude vs frequency
    - ii. Phase vs frequency
    - iii. Amplitude vs time (impulse response )
    - iv. Step response vs time
    - v. Impedance vs frequency
    - vi. Smith Chart impedance plots
    - vii. Filter transfer function plots
    - viii. Eye diagram plots

## Menu Views in SDLA

A few of the menus associated with each of the primary blocks in SDLA are shown in Figures 2-6. More extensive details are found in the on-line help manual for SDLA at [www.tektronix.com/sdla](http://www.tektronix.com/sdla).

The main menu in Figure 2 is simplified down into 4-basic blocks to represent the entire system.

The menu in Figure 3 allows a cascade of 4-port S-parameter blocks to be used to represent the model inside the embed block, or inside the de-embed block. The modeling handles all S-parameter terms for reflection, cross coupling and transmission.

The menu in Figure 4 represents the Tx model where the Thevenin equivalent circuit is used. The upper model is for the de-embedding from the acquired waveform back to the Thevenin voltage. The lower circuit is for re-applying that voltage into the simulation side with the embed blocks.

The menu in Figure 5 represents the Rx, receiver model. CTLE equalization can be set up, followed by clock recovery setup, followed by the FFE/DFE adaptive equalizer. This block is not modeled with S-parameters but rather with linear filtering, and non-linear FFE/DFE filtering, and AMI-IBIS models.

The menu Figure 6 represents a test point manager that allows the desired test point waveforms to be assigned to math waveform slots in the oscilloscope. It also allows the user to set up the overall bandwidth of the simulation results, and to control the delay of the filtered results.

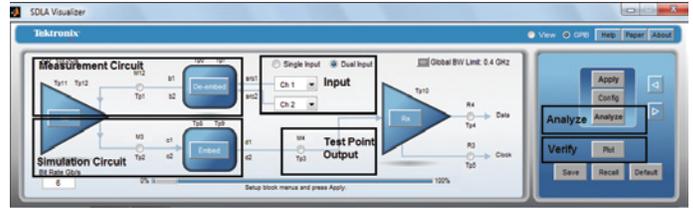


Figure 2. SDLA Visualizer main menu.

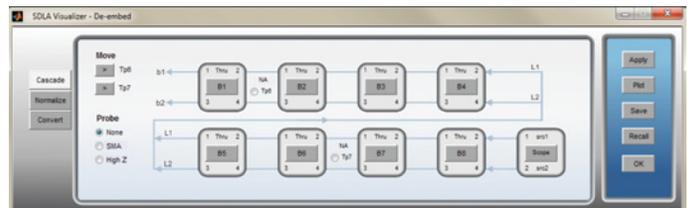


Figure 3. The De-embed cascade model diagram.

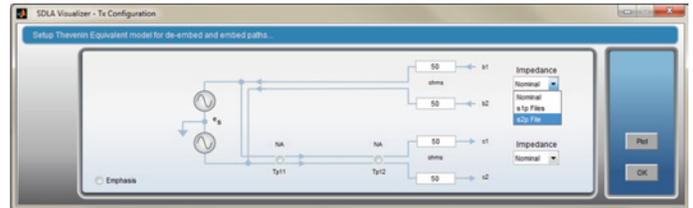


Figure 4. Tx model diagram.

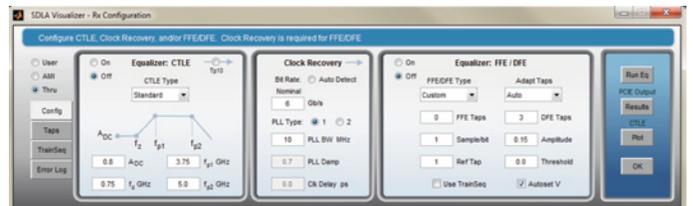


Figure 5. The Rx model configuration menu.



Figure 6. The test point manager setup menu.

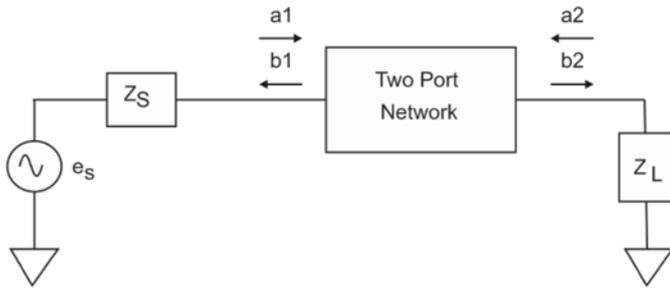


Figure 7. S-parameter two port network model.

## S-Parameter Modeling

High frequency design engineers commonly use the scattering parameters to mathematically model RF and microwave multi-port networks. Since SDLA represents model blocks with S-parameters it is important to have a good understanding of how they are defined, how they are measured, and how they are used and interpreted.

### S-parameter Definition

The basic two-port S-parameter model for a network is shown in Figure 7. Where  $Z_s$  is the source impedance of the generator that drives the network and  $Z_L$  is the load impedance attached to the output of the network. The signals into ports are represented by  $\mathbf{a}$  and the signals out of ports are represented by  $\mathbf{b}$ .

The value of  $a_1$  is the incident signal traveling into port one of the network. It is a sine wave signal having a specific frequency. S-parameters are measured over a range of frequencies. The value of  $b_1$  is the reflected signal from the input of the network due to impedance mismatch when port two is terminated with  $Z_L = Z_0$ . The value of  $Z_0$  is the reference impedance used during the measurement process of the S-parameters. The value is commonly 50 ohms. But it may be other values such as 75 ohms which is used in analog video systems. Likewise, if port one is terminated in  $Z_0$  and port two is driven by a generator then  $a_2$  is incident into port two then  $b_2$  would be the reflected signal. When considering transmission through the network  $a_1$  into port one may result in contribution to  $b_2$  out of port two. Likewise,  $a_2$  into port two may result in contribution to  $b_1$  out of port one.

Assume that  $E_{a_1}$  is a traveling wave into port one and that  $E_{b_1}$  is a traveling wave reflected back out from port one. Also, assume that  $E_{a_2}$  is a traveling wave into port two and that  $E_{b_2}$  is the reflected wave from port two.

The values of  $a$  and  $b$  may be characterized as follows and described in [6]. They will be referred to as traveling waves.

$$a_1 = \frac{E_{a1}}{\sqrt{Z_0}} \quad a_2 = \frac{E_{a2}}{\sqrt{Z_0}} \quad b_1 = \frac{E_{b1}}{\sqrt{Z_0}} \quad b_2 = \frac{E_{b2}}{\sqrt{Z_0}}$$

The S-parameters relate to the four waves in the equations above in the following manner:

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

The S-parameters are then defined as follows:

$$S_{11} = \frac{b_1}{a_1} \quad \text{when } a_2 = 0 \quad S_{12} = \frac{b_1}{a_2} \quad \text{when } a_1 = 0$$

$$S_{21} = \frac{b_2}{a_1} \quad \text{when } a_2 = 0 \quad S_{22} = \frac{b_2}{a_2} \quad \text{when } a_1 = 0$$

The value of  $a_1$  is zero when the input port one is terminated with  $Z_0$  and port two is driven with the test signal. The value of  $a_2$  is zero when the load impedance,  $Z_L$ , equals  $Z_0$  and port one is driven with the test signal. These are the conditions that a VNA or TDR system provide when measuring S-parameters.

The S-parameters are each a vector of complex numbers that are a function of frequency. The value of  $S_{11}$  and  $S_{22}$  are called reflection coefficients and  $S_{21}$  and  $S_{12}$  are called transmission coefficients.

The same procedure is applied for models with 3, 4 or more ports. For example, a signal is applied to any one port while all other ports are terminated in the reference impedance of  $Z_0$ . The dimensions of the resulting S-parameter matrix are (N, N, M) where N is the number of ports and M is the number of frequencies at which the measurements or calculations were made.

## Equations for Various Parameters using S-parameters:

### Reflection Coefficients:

The reflection coefficients into each port are  $S_{11}$  and  $S_{22}$  in a 2-port network. During the measurement process they have been normalized with respect to the reference impedance, and have a range of -1.0 to + 1.0 for both the real and the imaginary parts. Since it is the ratio of reflected / incident signal, a value of 1.0 represents all of the signal reflected due to an open circuit at the port input. A value of -1.0 is due to a short circuit which inverts the polarity of the incident signal when it is reflected.

**Impedance into the port** can be computed from the reflection coefficient by removing the normalization as follows:

$$Z = \frac{1 + \Gamma}{1 - \Gamma} \cdot Z_0$$

Where  $Z$  is the input impedance to the port,  $Z_0$  is the reference impedance, and  $\Gamma$  is the reflection coefficient of  $S_{11}$  or  $S_{22}$  for a 2-port network.  $Z$  and  $\Gamma$  are functions of frequency. The impedance may then be plotted as a function of frequency. A Smith chart is also commonly used to view the reflection coefficients in terms of normalized impedance, but cursors typically readout the actual impedance.

Re-arranging terms, the equation for normalizing the impedance to obtain the reflection coefficient can be written as:

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

**Return Loss** is another way in which the reflection coefficient may be represented in dB as a function of frequency. The equation most commonly used is:

$$returnLoss(f) = 20 \cdot \log_{10}(|\Gamma(f)|)$$

Where  $f$  is frequency and return Loss have units of dB.

### Magnitude or Phase vs. Frequency Equations

S-parameters are most commonly viewed graphically with units of magnitude in dB versus frequency. The equation for this is given as:

$$dB = 20 \cdot \log_{10}(|S_{mn}|)$$

$$phase = \tan^{-1} \left[ \frac{imag(S_{mn})}{real(S_{mn})} \right]$$

Where  $S$  is an S-parameter vector and  $m$  and  $n$  are indexes that identify which of the  $N \times N$  parameters it is. The value of  $N$  is the number of ports.

### Converting S-parameters to the Time Domain

The time domain is another useful view for analyzing the S-parameter data for correctness or to troubleshoot problems that can occur. The IFFT can be used to make this conversion, however, several mathematical steps must be performed before the IFFT function can be applied.

**The procedure for converting the S-parameter to the time domain is as follows:**

1. Extrapolate the S-parameter from the start frequency point back to DC. Since S-parameters are commonly obtained from a VNA measurement, the data set will not contain DC unless it was measured independently and added to the file.
2. Extrapolate or truncate to set the stop frequency to the Nyquist point at  $\frac{1}{2}$  the sample rate.
3. Create the data set for frequencies from Nyquist to the sample rate by copying the complex conjugate of the data from the first point after DC to Nyquist in reverse order. The details are not given here, however, this step requires intricate knowledge of the FFT. The copy is slightly different for an even number of points compared to an odd number for the IFFT length. For example if the IFFT has an even length then Nyquist will fall exactly on one of the samples in the frequency domain. If the IFFT length is odd then Nyquist will be half way between two sample points in the frequency domain. Any time an FFT is computed the frequency domain has both the data of interest from DC to Nyquist and a complex conjugate image of the same data from Nyquist to the sample rate. The last point in the FFT is one sample short of the sample rate value because the FFT is a circular function and the sample rate and DC are at the same point.
4. **Compute the IFFT of the resulting S-parameter data.** If this step is performed correctly then the IFFT of the S-parameter data will result in time domain data that is real only, with the imaginary part equal to zero.



Figure 8. An example of a cascade of 4-port S-parameter blocks for modeling the simulated link model.

## S-parameter Cascade

An example of a cascade of 4-port S-parameter blocks is shown in Figure 8. The circuit connections are defined by port numbers assignments. The S-parameter reference impedance for the ports that are connected must be the same in order for the cascaded results to be correct. The Normalize tab opens up a tool which can renormalize the reference impedance at any of the ports.

The 4-port magnitude plots ordered in the relative matrix positions provide a quick way to view the characteristics of a component, from a single view as shown in Figure 9.

- **The reflection coefficients**,  $S_{11}$ ,  $S_{22}$ ,  $S_{33}$ , and  $S_{44}$  are shown down the diagonal of the matrix. This is from the upper left corner down to the lower right corner.
- **The transmission terms**  $S_{21}$ ,  $S_{12}$ ,  $S_{34}$ , and  $S_{43}$  are on 2x2 sub matrix diagonals. One can identify them as transmission terms because the DC value starts close to 0 dB and slopes downward as frequency increases.
- **The remaining terms are cross coupling terms** between various ports. Typically at DC the cross coupling coefficients are close to 0 such that on a dB plot it is approaching minus infinity, which is the log of 0. This usually makes it readily distinguishable on the plots from the transmission terms.
- **For passive circuits** the forward and reverse transmission terms are identical. If they are not, then there are errors in the measurements. Even so, the reflection coefficient looking in from either end may be different. The SDLA Overlay plot can be used to compare. The user may choose which S-parameter plots to overlay in the graph.

The time domain impulse response representation of each of the above S-parameter vectors is shown in Figure 10. This is also a valuable view. It readily shows time delays of the transmission terms. It also indicates whether the data is settled over the time interval as it should be for a valid set of S-parameters. These plots are obtained by computing an IFFT

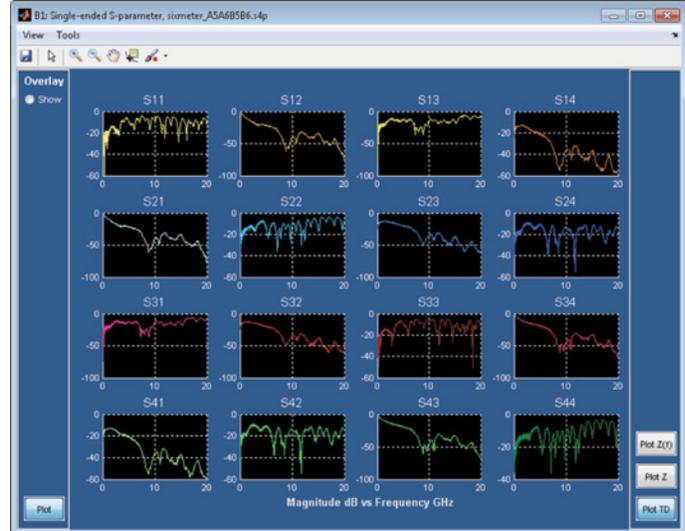


Figure 9. Frequency domain plots for a 4-port S-parameter set for a serial data cable pair and circuit board.

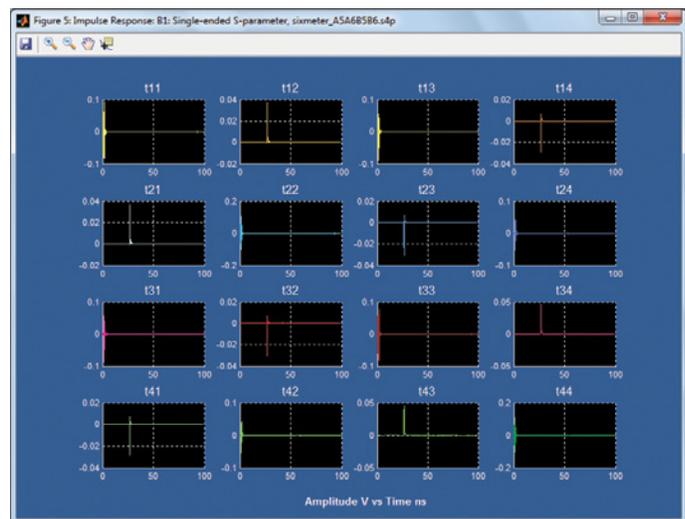


Figure 10. Time domain representation of a set of 4-port S-parameters for a data link.

of the frequency domain S-parameter data. This often requires extrapolating frequency domain data to DC, and sometimes requires extrapolation to a higher desired Nyquist frequency.

**Extrapolation to DC is often a problem area.** Small errors such as a 0.2 dB or 2 degrees in phase can cause significant tilt in the step response of filters built from that S-parameter set. The VNA, vector network analyzers cannot measure DC. TDR based S-parameter measurement instruments can measure down to DC.

## Measurement of S-parameters:

When a set of S-parameters is measured using a VNA, a sine wave incident signal is placed on a port. For reflection coefficients, the incident and reflected sine wave magnitude and phase are measured. All other ports must be terminated with the reference impedance. The ratio of the reflected and the incident signals is represented as  $S_{11}$ ,  $S_{22}$ ,  $S_{33}$ , or  $S_{44}$ . This is done for many frequencies. For transmission coefficients such as  $S_{21}$ , a sine wave is placed on Port 1 and measured on Port 2 and their ratio becomes  $S_{21}$ .

- This requires that the sine waves are at steady state after all reflections and transmissions have settled to steady state.
- Therefore, the sweep rate of the VNA must be slow enough such that all has settled prior to measurement at each frequency. Long cables require slower sweep rate than shorter devices such as PC board traces.
- Measure with TDR/TDT: S-parameters may also be measured and computed in the time domain using step generators for TDR or TDT responses. The step contains all frequencies simultaneously applied to the device that is being measured. Thus, all reflections and transmissions are contained in one step response for each signal.
- As with a VNA, the time interval acquired from the step must be long enough to the point where all reflections have settled.

## VNA Frequency Spacing and Time Response Duration

The frequency spacing of the VNA measured data will determine the number of samples up to the desired sample rate frequency at which the time domain waveforms are to be represented. The smaller the frequency spacing, the more frequency samples there are, and the longer the time interval will be. If the frequency spacing is too large and the resulting time interval is too short for the time domain data to be settled, then aliasing occurs. This results in the time domain signal being wrapped to incorrect positions. The equation to determine the time interval is given below:

$$T = 1 / \Delta f$$

Where T is the time interval covered by the S-parameter set, and  $\Delta f$  is the frequency spacing. This inverse relationship infers that in order to cover a longer T, the  $\Delta f$  needs to be smaller. This results in a finer frequency resolution which in turn results in a larger number of frequency domain samples up to the desired sample rate frequency.

The number of samples, N, may be computed from the following equation.

$$N = 0.5 fs / \Delta f$$

Where fs is the sample rate. The number of frequency domain samples covering the range of DC up to the sample rate frequency is equal to the number of time domain samples when the IFFT is computed to obtain the time domain response.

Therefore, the time interval is longer for a given sample rate when  $\Delta f$  is made smaller.

## Cascading S-parameters and Aliasing

The cascading of blocks represented by S-parameters is a key operation in the serial data link simulation and analysis system. To understand several of the issues involved consider the cascade shown in Figure 8. The model in each block is represented by a set of 4-port S-parameters. In order to compute transfer functions for the system test points, it is necessary to combine several cascaded blocks into a single block. Consider three blocks with 4 x 4 S-parameters matrices named B1, B2, and B3.

- These may be converted to T-parameter matrices. Then they may be multiplied together to obtain a single T-parameter matrix,  $T_T$ , for the total system. The  $T_T$  matrix may then be converted back to an S-parameter set. In order to combine the matrix sets they must be consistent:

Note: SDLA does not use T-parameter for cascading in order to avoid divide by zero issues that might arise. T-parameters were mentioned because they represent a more intuitive way to cascade by multiplying the matrices for each block together.

- Frequency spacing must be the same, thus time interval represented must be the same.
- Start and stop frequencies must be the same.
- Reference impedance must be the same for any two ports connected together.
- The final combined matrix will cover the same time interval of each block matrix. Thus if the system delay of the cascaded blocks is greater than the time interval covered by the individual block S-parameters then aliasing will occur. In the time domain the aliasing results in pulse response features occurring in the wrong time position and they may be reversed in time order. This is the result of phase aliasing in the frequency domain where there are less than 2 samples per revolution.

### An example:

To illustrate the problems involved consider the following example where three blocks are to be cascaded, and all of the blocks are the same. The  $S_{21}$  data in the frequency domain has been transformed to the time domain and has the impulse response shown in Figure 12. It is observed from the graph that the propagation delay is about 10ns. The S-parameter's frequency spacing of 50MHz covers a time duration of 20ns. It is sufficient for this  $S_{21}$  data set.

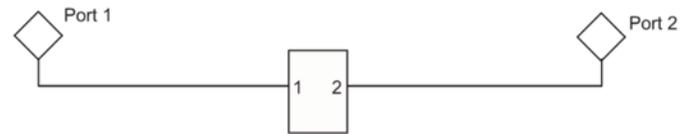


Figure 11. 2-port example model.

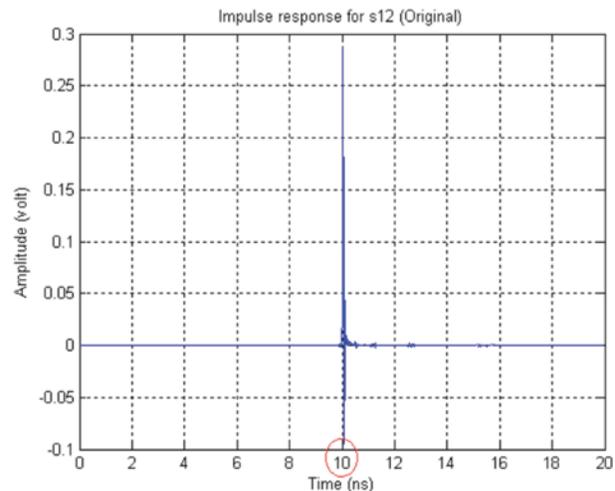


Figure 12. Impulse response of s12. The propagation delay is 10ns.

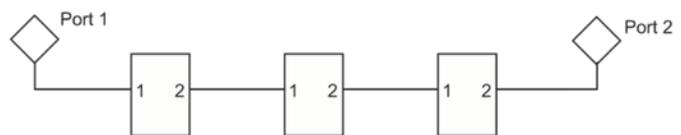


Figure 13. Consider 3 of these identical S-parameter blocks cascaded together.

However, when three of these S-parameter blocks have been cascaded, the total propagation delay would be 30ns. This would result in the impulse aliased to a delay of 10ns rather than the correct 30ns position. For this case, using the same frequency spacing is no longer sufficient to cover the combined blocks. This example demonstrates the first requirement to be addressed:

**Requirement 1:** Resample the individual S-parameters for each block to provide smaller frequency spacing to cover the increased time interval for the combined S-parameters. A plot showing the data set after resampling is shown in Figure 14.

Note: SDLA performs this resampling automatically.

Consider the single 2-port S-parameter model shown in Figure 13.

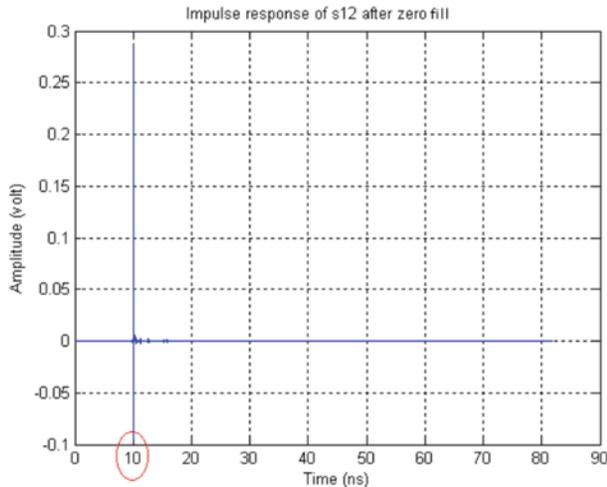


Figure 14. Impulse response of resampled s12. Covers longer time interval.

The plot shown in Figure 15 demonstrates that the interpolation does not significantly change the response of the S-parameter. However, always be wary of interpolation of the S-parameters. They are typically represented as 3-dimensional spirals that are a function of phase, magnitude, and frequency. Interpolation has the potential to introduce distortions to the higher rate of change areas on the curve and it has the potential to introduce ringing at the start and stop frequency areas of the data.

Various approaches can be taken to address Requirement 1. For example, one approach is to perform interpolation in the frequency domain. This may be done by either interpolating the real and imaginary components, or it may be done by interpolating the magnitude and phase components [3]. This approach can be done with linear interpolation but this can result in significant errors unless the frequency spacing is sufficiently small. Using higher order interpolation may result in transient errors at the start and stop frequencies where there is a discontinuity in the data set.

A different approach would be to resample multiple S-parameters that potentially have different frequency spacing and different bandwidth.

Note: A Tektronix patent disclosure has been submitted for this overall algorithm.

1. **Extrapolate all S-parameters data to DC** if the S-parameters don't have a DC value. S-parameters measured from a VNA do not have a DC value. TDR measured S-parameters have a DC value.



Figure 15. Magnitude responses of s12 before and after re-sample.

2. **Determine the common maximum frequency** for all of the S-parameter sets. This value can be the maximum frequency of all of the S-parameter sets in the cascade. Extrapolate each S-parameter set to beyond the maximum common frequency.
3. **Convert extrapolated frequency domain S-parameters** to obtain the time domain impulse responses using an IFFT.
4. **Determine the actual common sample period** between the impulse responses. The actual common sample period can be taken as the minimum of sample periods of the impulse responses. Then resample the impulse responses so they all have the same sample rate.
5. **Zero fill the impulse responses** at proper positions, as described below, to get increased time interval. The increased time interval can be determined as multiples of the sum of all the time intervals represented by each S-parameter.
6. **Convert time domain zero filled** impulses to frequency domain using FFT.
7. **Truncate the lower frequency and high frequency points** that are extrapolated. (This step is optional.)
8. **At this step all S-parameters have been resampled at the same frequency points with sufficient frequency resolution.** For each frequency point, combine the S-parameters for each block to be cascaded. Do this directly or with T-parameters to get the combined S-parameter.

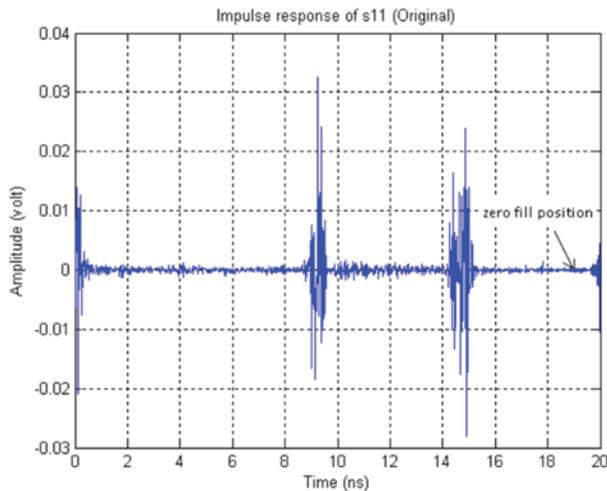


Figure 16. Impulse response of s11. Early ringing wrapped to the end.

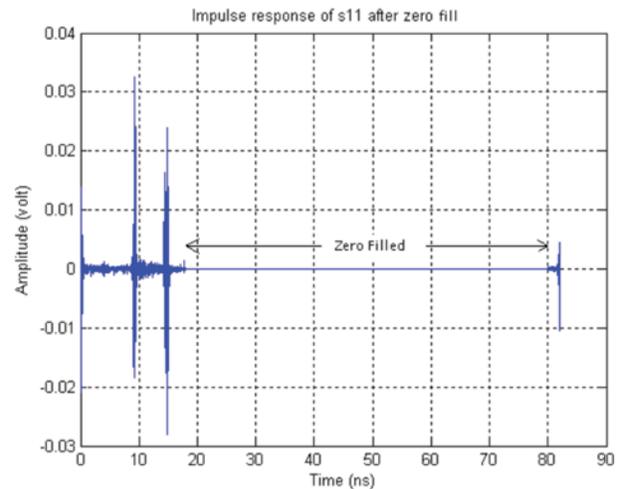


Figure 17. Impulse response of resampled s11. Early ringing preserved at the end.

## The Zero Fill Algorithms

In Step 5, the position of zero filling is not arbitrary and it is not necessarily started at the end of the right side of the time domain response.

For all of the impulse responses in the S-parameter set the, the zero phase time reference position is at the beginning of the time record, the left end. If the data were all ideal the zero fill would be added to the right side of the record. However, the leakage from the IFFT calculations can sometimes result in ringing from the beginning of the time record being wrapped to the end of the time record.

Therefore, the impulse responses obtained from converting to the time domain may have some ringing at the end of the time record. This ringing at the end the impulse response is caused by band limited nature of S-parameters and is affected by sample offset. For example, the impulse response for an  $S_{11}$  data obtained in Step 4 is shown in Figure 15. The small ringing at the end is wrapped from the left end to the right end. Regular zero padding, where zeros are padded to the right end of the impulse response would yield an S-parameter result with errors. This example demonstrates the second issue to be addressed:

**Requirement 2:** Zeros are filled at proper position to maintain the wrapped ringing at the end of the impulse response.

To address requirement 2, there are two options to find the proper position:

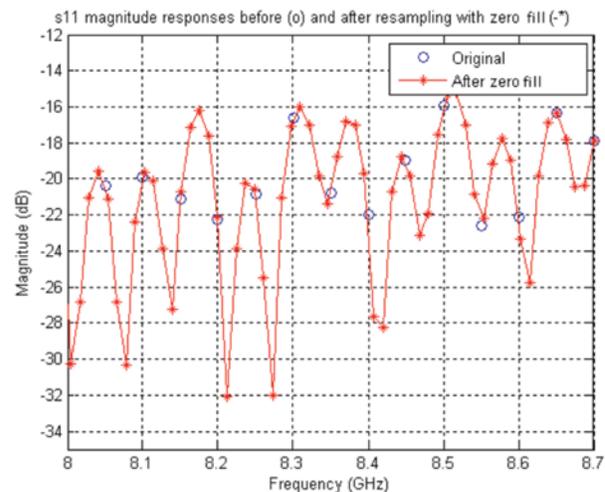


Figure 18. Zoomed in magnitude responses of s11 before and after re-sample.

**Option 1:** Starting from the end the impulse response, check if there is ringing at the end. If there is no ringing, then the zeros can be filled right after the last point of the impulse response. If there is ringing, then search backwards to find the position where the ringing is settled. Zeros can be filled in the settled position as shown in Figure 16.

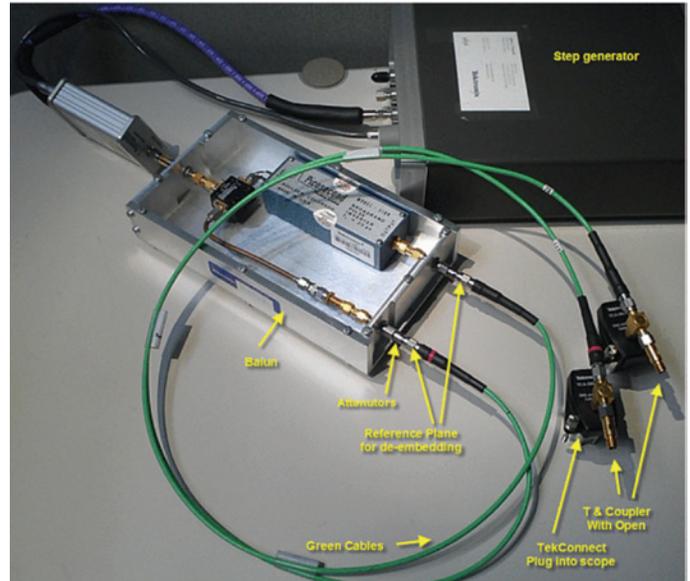
**Option 2:** Always pick a certain percentage of the impulse response to fill zeros. For example, zero fill at a position of 5% from the end. Figure 17 and Figure 18 show the results of Option 2. The resampled S-parameters are matched well with the original S-parameters.

## De-embedding Example with a Large Reflection

This section provides a dramatic example of using S-parameter models to perform de-embedding and embedding. The resulting waveforms are live on an oscilloscope.

In Figure 19, a step generator is driving a balun to provide a differential step signal. Two 5X attenuators are connected to the two outputs of the balun where the green cables are connected. These insure minimum reflections from the generator at the reference plane for de-embedding. In order to provide a large reflection for the purpose of this example, the opposite ends of the green cables are each connected to a T and coupler combination with an open circuit.

A VNA was used to measure 2-port S-parameters for each green cable and the two T's combined with open circuit couplers. These 2-port S-parameter sets for the green cables can be loaded into block 1 of the cascade and treated as a 4-port model. The 2-port S-parameters for the two T's can be loaded into block 2 of the cascade. These blocks are in the de-embed system block to define the measurement circuit into the scope. Nominal 50 ohms is defined as the scope impedance and for the balun combined with the step generator for this example.



**Figure 19.** Test setup for de-embedding the green cables and a T connector with an open circuit coupler.

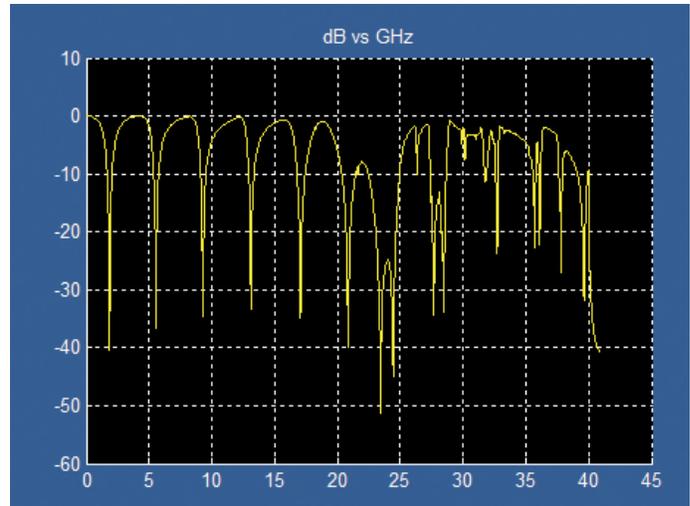
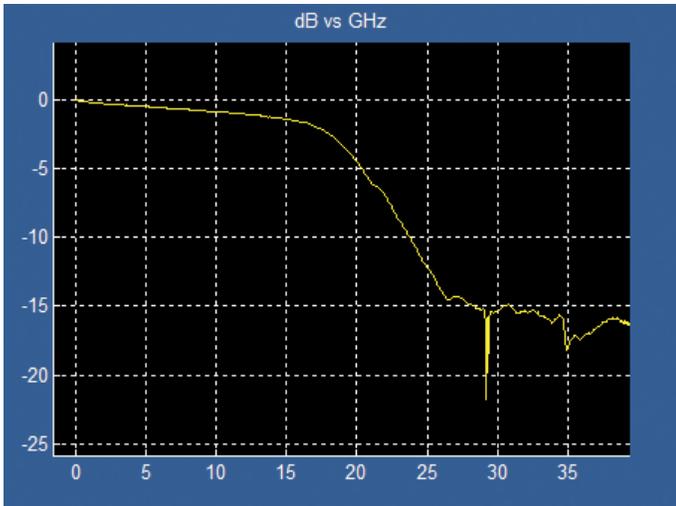


Figure 20. S21 for one the green cable is shown at the left. S21 for one T and coupler is shown at the right.

The green cables have relatively small loss over the range of 10GHz as shown in Figure 20. However, the T with open circuit coupler has dropouts as deep as -40 dB in the same range! This is a difficult de-embedding situation since the transfer function will end up with inverse response of up to 40 dB peaks. The oscilloscope will be operated in average acquisition mode for this example in order to increase the bits of resolution and eliminate noise from the large boost.

Figure 21 shows the **Tp1** waveforms, which are the outputs of the balun with the cables and T coupler combination still loading the step generator. The waveforms acquired on the scope are in yellow and Cyan. The waveforms de-embedded to the reference plane with the cables still loading that point are shown in purple and orange.

The resulting reflections and time delays are correctly represented in the de-embedded waveforms.

The reflections due to the open circuit of the coupler can be seen on the acquired waveform at the rising and falling edges where the steps entered the oscilloscope (yellow and cyan). On the de-embedded waveforms (purple and orange), the rising and falling edges have the reflections removed, and

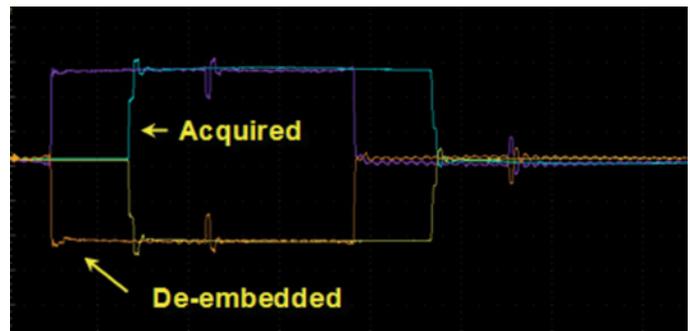


Figure 21. De-embedded waveforms with cables and T's connected to the balun.

the shape of the pulse is as expected at the reference plane into the cables. There is a delay of approximately 4.1 ns through the cable. At 8.2 ns, the round trip time of 8.2 ns, the reflection arrives back at the reference plane input to the green cables. Also, when the pulses (purple and orange) go back to zero, another round trip reflection off of the input side to the scope arrives back to the reference plane 8.2 ns later. See Figure 21.

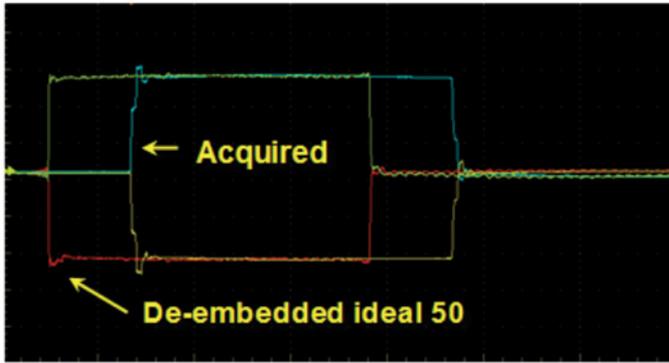


Figure 22. De-embedded waveforms with cables and T's replaced by ideal 50 ohms, at Tp2 as shown in Figure 2.

Figure 22 shows **Tp2**, which provides the waveforms with the measured cables and components removed, and the step generator reference plane terminated in an ideal 50 ohms. This means all of the 4-port S-parameter blocks in the Embed block in the main SDLA menu were set to ideal Thru and the termination load at the end of the cascade was set for ideal 50 ohms. Thus this replaces the actual circuit that had the cables and the T connectors.

The wave-shape in Figure 22 is the expected shape of the step generator. One step went through a transformer and the other through a rigid cable in the balun. Thus they have different shapes as recovered through the S-parameter model simulation. The major reflections on the acquired waveforms were removed.

The transfer functions created from the S-parameter models, are FIR filters that run in real-time on the oscilloscope. The plots of these filters for the Tp1 test point illustrate the extreme nature of this particular de-embed example. The filter has close to 40dB peaks in its passband. As mentioned above, the scope must be put in average mode to accommodate removing noise from this extreme boost. This is a case that many would have thought was impossible to implement on an 8-bit oscilloscope, however, it is made possible because averaging of the waveform acquisitions results in extended vertical resolution, and it removes noise.

Note that the impulse response of this example takes a very long time to settle as shown in Figure 23.

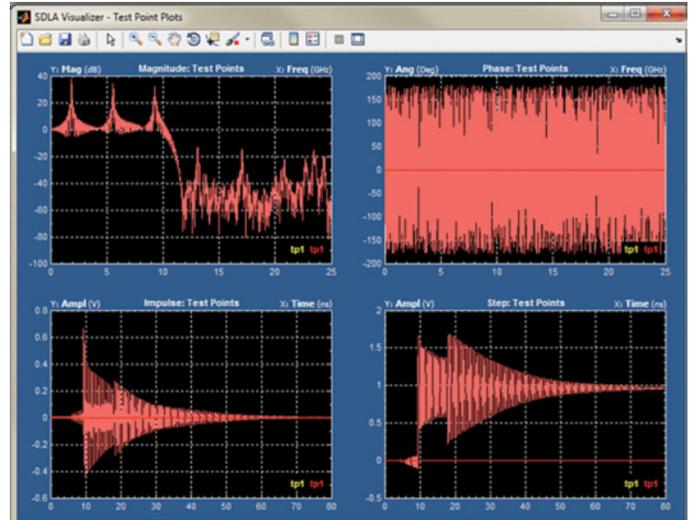


Figure 23. FIR filter transfer functions for the de-embedding example.

## Embed Simulation Model Overview

The embed simulation model consists of Tx, Embed cascade, and Rx input load setup as S-parameter model blocks. The cascade model blocks are 4-port S-parameter blocks that may be defined by numerous configurations with an additional set of model menus. All of these models are represented at the cascade level as 4-port S-parameter blocks. The Thevenin equivalent voltage source in the Tx block drives the cascade with a signal for each of two lines A and B. This voltage was computed by SDLA by making de-embed transfer functions to apply to the live acquired input waveforms from the oscilloscope.

## Summary of SDLA Process to Create the Embed Filter

1. Obtain the S-parameters for the channel block(s).
2. Extract the appropriate S-parameter transmission response.
  - If the S-parameter start frequency does not include DC or if the start frequency is not equal to the spacing then extrapolate to DC and also interpolate and resample if necessary to insure that frequency samples are an integer multiple of the sample rate. Also, if S-parameter data does not go up to Nyquist then extend it by point replication of last point.
3. Apply a band limiting filter by multiplying its frequency domain response with the S-parameter transmission response. If the S-parameter data had been point replicated to Nyquist then this helps minimize the effect of the discontinuity by more smoothly attenuating it to a low value.
4. Create a complex conjugate set of the S-parameter from DC plus one frequency interval to Nyquist mirrored around Nyquist.
5. Compute an IFFT to obtain the impulse response of the embed filter.
6. Remove leading zeros and shorten both ends of the response by an amount that still yields sufficient accuracy.

**The embed filter is computed as follows:**

$$h_{embed} = IFFT(H_{TF} \cdot H_{bw}) \quad (1)$$

Where  $H_{TF}$  is the transfer function of the DUT and  $H_{bw}$  is a symmetrical linear phase bandwidth limit filter. This transfer function is applied to the acquired input waveform on the oscilloscope to obtain a waveform representative of a selected test point within the SDLA S-parameter modeled network.

## S-parameter Extrapolation to DC and Nyquist

If the S-parameter data set for the Fixture or the Channel does not go all the way to DC nor all the way to the Nyquist frequency at which the filters will operate then SDLA will extrapolate the S-parameter set.

Extrapolation or interpolation of S-parameters is an area that can be difficult to deal with and sometimes may result in significant errors. Therefore, it is best to measure the S-parameters to DC and measure them all the way to the Nyquist frequency.

If the S-parameters are measured using a TDR system then DC is included in the resulting S-parameter data set. However, if a VNA is used to measure the S-parameters then DC is not included unless the user measured it with a volt meter or extrapolated it.

Since the filters computed in SDLA are typically of long length, a small magnitude error of a few tenths of a dB in the extrapolated samples to DC can result in a relatively large error in the step response of the filtered data. This is because the small gain error is accumulated over the full length of the filter during convolution when the filter is applied to a waveform. A small phase error, such as 2 degrees, in this extrapolated data, can introduce a large error in the step response.

**SDLA extrapolates to DC** by taking the magnitude of the first two points in the S-parameter data and computing a linear line of equal slope back to DC. This 1st order approach is superior to applications that point replicate the 1st point back to DC. Point replication would be more likely to create significant error in the step response of the final filter. Especially when there are several points that need to be extrapolated. The linear extrapolation typically works well for  $S_{21}$  data as long as the data is smooth and the DUT has close to a 1st order response in that region.

However, reflection coefficient data, for  $S_{11}$ ,  $S_{22}$ , etc. typically spirals as it approaches DC and the linear extrapolation can easily have large error.

The phase response is unwrapped and the first two points are used to create an equal slope back to DC value. Phase at DC may only be 0 or 180 degrees. Thus both phase and magnitude are handled.

**The user may check the extrapolation** accuracy in SDLA by using the plots in various menus where the S-parameter data is loaded. Use the zoom tool to look closely at the magnitude extrapolation to DC. The magnitude response should look continuous without a discontinuity. If it looks okay then continue on. However, if the result does not look okay then either measure the points or manually extrapolate the data and insert it into the Touchstone file. However, for SDLA Visualizer, all of the S-parameter data is used for computing the transfer functions. This means that all of them must be extrapolated. Keeping them all consistent in phase and magnitude with respect to each other would be difficult and therefore, manual editing is not a recommended procedure. But if the data is such that some of the errors are minimal, then improved results may be obtained with manual editing.

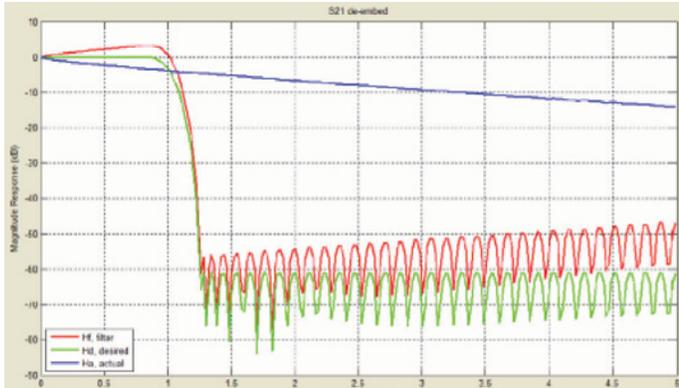


Figure 24. Example of creating a de-embed filter.

## De-embed Filter Derivation Overview

The purpose of this section is to provide a general visual concept of what the transfer functions for de-embedding look like. Shown in Figure 24 is a frequency domain plot of S-parameter transmission characteristic for a cable. This is shown as the blue trace. If it were desired to de-embed the cable then what is needed is to create a filter that has the inverse response as shown in red. Thus if the filter were applied to signal at the output of the cable then the response would appear as shown with the green desired plot.

### Step-by-step Procedure to Derive a De-embed Filter

A more detailed graphical representation of the step-by-step procedure to derive the test point filters is shown in Figure 25.

1. Obtain the S-parameters for the blocks in the system that represents the measurement path and the simulation path.

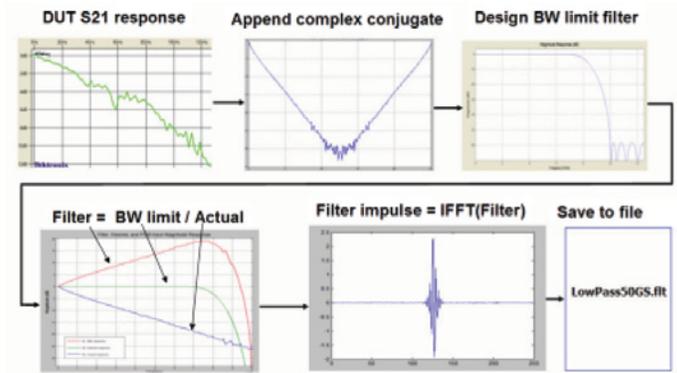


Figure 25. The test point filter creation process.

2. If the S-parameter start frequency does not include DC or if the start frequency is not equal to the spacing then extrapolate to DC and interpolate and resample if necessary to insure that frequency samples are an integer multiple of the sample rate. If the S-parameter data does not go up to Nyquist, then extend it by point replication of last.
3. Extract the appropriate S-parameter transfer function response. The transfer function multiplied times the input waveform in the frequency domain will result in the waveform at the desired test point. This will be different for each test point in the system.
4. Multiply the result in the frequency domain by a bandwidth limit filter. SDLA may compute this automatically or the user may override it and specify a custom filter.
5. Create a complex conjugate set of the S-parameter from DC to Nyquist mirrored around Nyquist.
6. Compute an IFFT to obtain the impulse response of the filter.
7. Remove leading zeros and shorten the length of the filter by truncating the response below a threshold.

## Causality

The bandwidth limit filters used by SDLA are linear phase symmetrical filters. They are non-causal because they have been implemented with zero delay. Causal bandwidth limit filters that exhibit no pre-shoot on an edge were not used because they would have resulted in a non-linear phase response in the transition band of the filter. This in turn could result in a modeled eye diagram that would be tilted due to the high frequency phase distortion. Also, the causal filter with no pre-shoot has more overshoot following the transition. That would make it harder for an eye to pass a mask test. The use of linear phase, symmetric, non-causal, bandwidth limit filters avoids these problems resulting in a more accurate eye diagram representation. If the bandwidth of the signal is lower than the bandwidth of the filter then non-casual ringing will not be seen.

## Time Domain Wrapping of De-embed Filter Associated With Delay

If a de-embed filter is derived from S-parameters the resulting impulse response will likely be wrapped in the time domain. Therefore, an algorithm is incorporated into SDLA to prevent wrapping from occurring.

The following example will help illustrate the concept. The computation of a de-embed filter requires computing the inverse frequency response which represents division as follows:

$$H_{deEmbed} = \frac{|H_{bw}| \angle \theta_{bw}}{|H_a| \angle \theta_a} \quad (3)$$

Where  $H_{deEmbed}$  is frequency domain response of the de-embed filter and  $H_{bw}$  is the bandwidth limit filter response and  $H_a$  is the actual system response that is to be de-embedded. By using the rules for polar number division the above equation may be rewritten as:

$$H_{deEmbed} = \frac{|H_{bw}|}{|H_a|} \angle (\theta_{bw} - \theta_a) \quad (4)$$

This indicates that the phase of the DUT represented by  $\theta_a$  is now negative. Thus the time domain impulse response of the de-embedded result from the DUT will have negative delay. This is expected since it takes a positive delay for the signal to travel through the DUT in the first place.

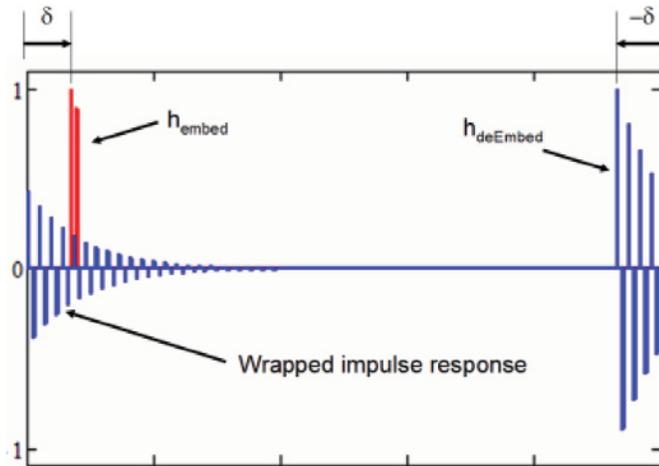
Wrapping of the filter time domain impulse response can occur as a result of the negative delay from de-embed filter computation. The time domain response,  $h_{embed}$ , for the embed filter can be computed from:

$$h_{embed} = IFFT(H_{bw} \cdot H_a) \quad (5)$$

Likewise, the time domain response for the de-embed filter can be computed as:

$$h_{deEmbed} = IFFT(H_{bw} \setminus H_a) \quad (6)$$

This assumes the complex conjugate part of H data set was created from Nyquist to the sample rate frequency prior to computing the IFFT.



**Figure 26.** Example of how an inverse de-embed function results in wrapping of the time domain impulse.

The IFFT is a circular function. If the original DUT delay was  $\delta$  seconds, then the impulse response computed from the S-parameters of the DUT would occur  $\delta$  seconds from the left hand side of the time record. However, if one computed the IFFT of  $1/H_a$  then the delay becomes negative and the impulse response of the result would start at  $-\delta$  seconds from the right hand side of the time record. For this case if the impulse response had not settled within the period of  $\delta$  seconds then the remainder of it would be wrapped starting at the left side of the time record as shown in Figure 26.

Consider the hypothetical example shown in Figure 26 where the red trace is  $H_a$  in the time domain and the blue trace is  $\text{IFFT}(1/H_a)$ . This is an example of the wrapped impulse response. SDLA has an internal algorithm to prevent this from occurring.

However, if the DUT S-parameters had been measured with insufficient number of points such that the resulting time interval is less than the length of the impulse response then this would result in potentially having the impulse response wrapped over the top of itself multiple times. For this case SDLA cannot unwrap the filter and the user would have to re-measure the S-parameters.

Also, notice that even though the red trace of  $H_a$  has a short time duration the inverse response represented by  $\text{IFFT}(1/H_a)$  can actually cover a much longer time interval depending on the content of the  $H_a$ .

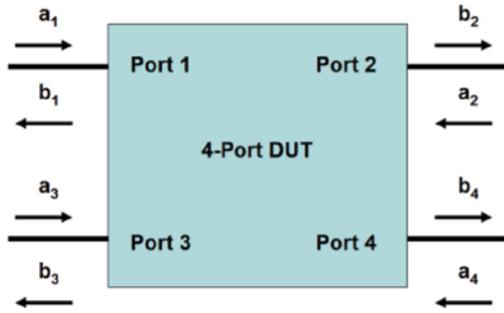


Figure 27. Four port DUT and input and output signal notation.

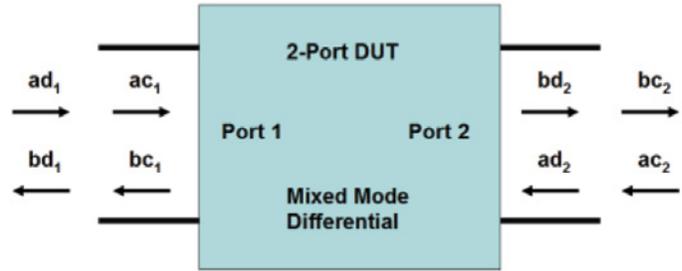


Figure 28. Representation as a 2-port mixed mode DUT with differential Port 1 and differential Port 2.

### Four Port S-parameter Single-ended to Differential Mixed Mode Conversion

This section covers the math derivation needed to understand how to convert a standard four port S-parameter matrix into a differential mixed mode representation. The standard definition matrix assumes four single-ended ports with S-parameters organized as shown in Figure 27 and equation (1).

$$S_{std} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{21} & S_{22} & S_{23} & S_{24} \\ S_{31} & S_{32} & S_{33} & S_{34} \\ S_{41} & S_{42} & S_{43} & S_{44} \end{bmatrix} \quad (1)$$

Where  $S_{std}$  is the standard S-parameter matrix layout with a and b variables that represent the single ended input and output signals respectively at each port.

In many cases a preferable method of interpreting the S-parameters would be to view the DUT as a 2-port mixed mode differential device rather than four port single ended device. The drawing in Figure 28 shows such a layout.

In Figure 28 the **a** and **b** variables have a **d** added to indicate differential and they still represent incident and reflected voltages respectively. The relationship of the **a** and **b** variables from the previous figure with respect to the **ad** and **bd** variables of the mixed mode example are as follows:

$$ad_1 = \frac{1}{\sqrt{2}} \cdot (a_1 - a_3) \quad (2) \quad ad_2 = \frac{1}{\sqrt{2}} \cdot (a_2 - a_4) \quad (3)$$

$$ac_1 = \frac{1}{\sqrt{2}} \cdot (a_1 + a_3) \quad (4) \quad ac_2 = \frac{1}{\sqrt{2}} \cdot (a_2 + a_4) \quad (5)$$

$$bd_1 = \frac{1}{\sqrt{2}} \cdot (b_1 - b_3) \quad (6) \quad bd_2 = \frac{1}{\sqrt{2}} \cdot (b_2 - b_4) \quad (7)$$

$$bc_1 = \frac{1}{\sqrt{2}} \cdot (b_1 + b_3) \quad (8) \quad bc_2 = \frac{1}{\sqrt{2}} \cdot (b_2 + b_4) \quad (9)$$

Now assume that the following equation may be written to convert the standard single ended input signals,  $A_{std}$ , into the differential and common mode matrix,  $A_{mm}$ , by multiplying it times a conversion matrix,  $M$ .

$$A_{mm} = M \cdot A_{std} \quad (10)$$

$$A_{std} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad (11)$$

$$A_{mm} = \begin{bmatrix} ad_1 \\ ad_2 \\ ac_1 \\ ac_2 \end{bmatrix} \quad (12)$$

Expand (10) as follows by filling in the appropriate values in M to represent equations (2), (3), (4), and (5) in matrix form:

$$\begin{bmatrix} ad_1 \\ ad_2 \\ ac_1 \\ ac_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{bmatrix} \quad (13)$$

Thus the value of M is shown in (3):

$$M = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad (14)$$

Now perform a similar derivation for the output signals. It turns out that the equations will map such that M is still the same as shown in (13):

$$B_{mm} = M \cdot B_{std} \quad (15)$$

$$B_{std} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \quad (16)$$

$$B_{mm} = \begin{bmatrix} bd_1 \\ bd_2 \\ bc_1 \\ bc_2 \end{bmatrix} \quad (17)$$

Use the equations in (2) through (9) to map the following equation:

$$\begin{bmatrix} bd_1 \\ bd_2 \\ bc_1 \\ bc_2 \end{bmatrix} = \frac{1}{\sqrt{2}} \cdot \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} \quad (18)$$

Now write an equation that relates the differential mixed mode S-parameters to the differential input and output voltages:

$$B_{mm} = S_{mm} \cdot A_{mm} \quad (19)$$

Substituting values into (19) illustrates the relationship of parameters and also shows the names assigned to the differential mixed mode S-parameters.

$$\begin{bmatrix} bd_1 \\ bd_2 \\ bc_1 \\ bc_2 \end{bmatrix} = \begin{bmatrix} S_{dd11} & S_{dd12} & S_{dc11} & S_{dc12} \\ S_{dd21} & S_{dd22} & S_{dc21} & S_{dc22} \\ S_{cd11} & S_{cd12} & S_{cc11} & S_{cc12} \\ S_{cd21} & S_{cd22} & S_{cc21} & S_{cc22} \end{bmatrix} \cdot \begin{bmatrix} ad_1 \\ ad_2 \\ ac_1 \\ ac_2 \end{bmatrix} \quad (20)$$

where  $B_{mm}$  is the mixed mode output voltage vector and  $A_{mm}$  is the mixed mode input signal vector, and  $S_{mm}$  is the mixed mode S-parameter matrix. The solution for  $S_{mm}$  may be written as:

$$S_{mm} = M \cdot S_{std} \cdot M^{-1} \quad (21)$$

where  $S_{mm}$  is the mixed mode S-parameter matrix and  $S_{std}$  is the single ended S-parameter matrix. Therefore, (21) is used to compute the mixed mode S-parameter matrix.

This results in  $S_{mm}$  equal to the following in terms of the original single ended S-parameters:

$$S_{mm} = \frac{1}{2} \cdot \begin{bmatrix} S_{11} - S_{31} - S_{13} + S_{33} & S_{12} - S_{32} - S_{14} + S_{34} & S_{11} - S_{31} + S_{13} - S_{33} & S_{12} - S_{32} + S_{14} - S_{34} \\ S_{21} - S_{41} - S_{23} + S_{43} & S_{22} - S_{42} - S_{24} + S_{44} & S_{21} - S_{41} + S_{23} - S_{43} & S_{22} - S_{42} + S_{24} - S_{44} \\ S_{11} + S_{31} - S_{13} - S_{33} & S_{12} + S_{32} - S_{14} - S_{34} & S_{11} + S_{31} + S_{13} + S_{33} & S_{12} + S_{32} + S_{14} + S_{34} \\ S_{21} + S_{41} - S_{23} - S_{43} & S_{22} + S_{42} - S_{24} - S_{44} & S_{21} + S_{41} + S_{23} + S_{43} & S_{22} + S_{42} + S_{24} + S_{44} \end{bmatrix} \quad (22)$$

The labels given to the elements of  $S_{mm}$  are as indicated below in (23) :

$$S_{mm} = \begin{bmatrix} S_{dd11} & S_{dd12} & S_{dc11} & S_{dc12} \\ S_{dd21} & S_{dd22} & S_{dc21} & S_{dc22} \\ S_{cd11} & S_{cd12} & S_{cc11} & S_{cc12} \\ S_{cd21} & S_{cd22} & S_{cc21} & S_{cc22} \end{bmatrix} \quad (23)$$

The  $S_{ddxx}$  values represent the two port differential signals. The  $S_{ccxx}$  values represent the two port common mode signals. The  $S_{dcxx}$  and  $S_{cdxx}$  represent cross talk terms.

SDLA will use  $S_{dd21}$  for deriving the FIR filter for embedding or de-embedding. As can be seen (24) the transmission term of each line is used. In addition the transmission term from the input of one line to the output of the other line is also taken into account. All four terms tend to have significant effects on differential cables with twisted pairs.

$$S_{dd21} = 0.5 \cdot (S_{21} - S_{41} - S_{23} + S_{43}) \quad (24)$$

SDLA allows the user to specify an alternate notation which will be used for interpreting where  $S_{dd21}$  is located in the Touchstone file. This is shown in (25). The next section explains how the position of  $S_{dd21}$  for alternate mode relates to the previous derivation equations.

$$S_{mm} = \begin{bmatrix} S_{cc11} & S_{cc12} & S_{cd11} & S_{cd12} \\ S_{cc21} & S_{cc22} & S_{cd21} & S_{cd22} \\ S_{dc11} & S_{dc12} & S_{dd11} & S_{dd12} \\ S_{dc21} & S_{dc22} & S_{dd21} & S_{dd22} \end{bmatrix} \quad (24)$$

## Position of $S_{dd21}$ in a Mixed Mode Matrix

An important reason why the mixed mode derivation is presented in this design document is to analyze what factors determine the position of  $S_{dd21}$  in the final  $S_{mm}$  mixed mode matrix.

Port number assignment as shown in Figure 28 does not affect the position of  $S_{dd21}$  in the final  $S_{mm}$  matrix.

However, the order of  $bd_1$ ,  $bd_2$ ,  $bc_1$  and  $bc_2$  and the order of  $ad_1$ ,  $ad_2$ ,  $ac_1$ , and  $ac_2$  in equation (20) are what determine where  $S_{dd21}$  resides in the  $S_{mm}$  matrix.

## Renormalization of S-parameters to a Different Reference Impedance

SDLA Visualizer has a **Normalize** menu in the de-embed and embed cascade menus. This allows the user to open up a set of 4-port S-parameters and change the reference impedance. This is useful if the S-parameter set has reference impedance that is not 50 ohms as required for cascading connections in these menus.

Details for the use of this tool can be found in the on line help manual for SDLA.

This section covers the mathematic derivations needed to renormalize a set of S-parameters.

### Equations to Perform Renormalization for a 4-port Network

The following transformation equations were derived by Laudie Doubrava using references [1], [2], and [3].

- Let the original reference impedance of port 1 be  $za1$ .
- Let the original reference impedance of port 2 be  $za2$ .
- Let the original reference impedance of port 3 be  $za3$ .
- Let the original reference impedance of port 4 be  $za4$ .
- Let the new reference impedance of port 1 be  $zb1$ .
- Let the new reference impedance of port 2 be  $zb2$ .
- Let the new reference impedance of port 3 be  $zb3$ .
- Let the new reference impedance of port 4 be  $zb4$ .

Then:

$$Za = \begin{bmatrix} za1 & 0 & 0 & 0 \\ 0 & za2 & 0 & 0 \\ 0 & 0 & za3 & 0 \\ 0 & 0 & 0 & za4 \end{bmatrix} \quad (1)$$

Where  $Za$  is the matrix containing the reference impedance for each port of the device S-parameters,  $S$ .

$$N1 = \begin{bmatrix} \frac{1}{\sqrt{za1}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{za2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{za3}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{za4}} \end{bmatrix} \quad (2)$$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Convert the S-parameters to Z-parameters:

$$Z = (I - N1^{-1} \cdot S \cdot N1)^{-1} \cdot (I + N1^{-1} \cdot S \cdot N1) \cdot Za \quad (4)$$

The matrix,  $S$ , represents the S-parameters of the device to be renormalized.

Define the new reference impedance matrix for each port 1 through 4:

$$Zb = \begin{bmatrix} zb1 & 0 & 0 & 0 \\ 0 & zb2 & 0 & 0 \\ 0 & 0 & zb3 & 0 \\ 0 & 0 & 0 & zb4 \end{bmatrix} \quad (5)$$

$$N2 = \begin{bmatrix} \frac{1}{\sqrt{zb1}} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{zb2}} & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{zb3}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sqrt{zb4}} \end{bmatrix} \quad (6)$$

Convert the 4-port matrix,  $Z4$  into a 4-port S-parameter matrix,  $SR$  which has the new reference impedance at each port of  $zb1$ ,  $zb2$ ,  $zb3$ , and  $zb4$ .

$$SR = N2 \cdot [(Z - Zb) \cdot (Z + Zb)^{-1}] \cdot N2^{-1} \quad (7)$$

## References

- [1] Ross A. Speciale & John C. Tippet, "A Rigorous Technique for Measuring the Scattering Matrix of a Multiport Device with a Two-Port Network Analyzer", IEEE Trans. Microwave Theory and Tech, Vol. MTT-30, No.5, p661-666, May 1982.
- [2] H. Dropkin, Comments on "A Rigorous Technique for Measuring the Scattering Matrix of a Multiport Device with a Two-Port Network Analyzer", IEEE Trans. Microwave Theory and Tech, Vol. MTT-31, No. 1, p79-81, January 1983.
- [3] Robert J. Weber , "Introduction to Microwave Circuits Radio Frequency and Design Applications", Wiley-IEEE Press; 1 edition, January 2001.
- [4] Contributors to this application: Pavel Zivny, Kan Tan, John Pickerd, Maria Agoston, Steve Corey, Kalev Sepp, Randy White, Dorine Gurney
- [5] Author of this paper: John Pickerd, Kan Tan, Tektronix, Inc.
- [6] "S-parameter design", Agilent Technologies Application Note, 1972

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